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ESTIMATION OF THE TIME FOR PRESSURE GROWTH IN A TANK WITH A CRYOGENIC LIQUID IN ZERO GRAVITY

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Two calculation models for estimating the rate of pressure growth in a large nondrained tank filled with a liquified gas under zero gravity are analyzed. Two limiting cases are considered. According to the first model it is assumed that heat is distributed uniformly over the entire mass of the liquid and gas, which are in thermodynamic equilibrium. This model yields the upper bound of the time in which a prescribed pressure is attained in the tank. In the second model one completely ignores any convection, the liquid boils at the tank wall, and the time is estimated (the lower bound) according to a nonstationary ablation model.

Development of space engineering poses the problem of long-term storage of large amounts of liquid in tanks under zero gravity. An example of long-term storage is a fuel resupply station for space vehicles whose design is expected to be implemented in outer space by 1997 [1]. The long stay of large vessels with liquids, especially low-boiling ones, in space poses a series of interesting problems. One of these problems is the development of a simple method for estimating the pressure growth in a tank until the safety valve starts to operate as a result of lengthy inflow of small amounts of heat to a low-boiling component in the tank. It is of primary importance to have a method for this estimation in the case of storage of liquid hydrogen in a tank in view of its very low boiling temperature. The heat flux that reaches the liquid is governed by the design and properties of the heat insulation and the presence or absence of means of active thermostatic control. Problems of ensuring heat insulation of the tank are not considered in the present work. The objective of the work is to obtain methods for estimating the pressure growth in the tank as a function of the value q of the heat flux that reaches directly the liquid boundary.

The main difficulty of this calculation is that the character of the motion of the large liquid mass in the tank, governing its heating and evaporation, is not determined. Since the tank with the liquid is in the zero gravity state the appearance of natural convection in the field of mass forces is hindered. However, the nonuniform surface tension due to nonuniform heating of the liquid serves as a driving force of convection. It is well known that a film of the gas-liquid interface moves in the direction of decreasing temperature and entrains the liquid, changing the temperature field in it. If initially there is a gas bubble in the tank and the tank wall temperature is above the liquid temperature, the bubble will be attracted to the nearest warmer portion of the tank wall. The motion of the gas bubble will mix a portion of the liquid, and the bubble itself will split into smaller ones under certain conditions (for example, with small loads that occur in some regimes of a space flight) [2].

Clearly, it is not possible to calculate the heat transfer under these poorly defined conditions. The present work proposes a model approach to solving the stated problem. This approach consists in estimating heating and evaporation of the liquid in two limiting cases: 1) heating and evaporation of the liquid are calculated under the assumption of complete mixing and 2) heating and evaporation of a stationary liquid at the tank walls are calculated with total neglect of convection. Similar assumptions were used in [3] to calculate the increase in pressure in tanks containing low-boiling liquids. This method yields lower and upper bounds of evaporation times of the liquid in zero gravity.

Model of Complete Mixing. Let the liquid temperature be T_0 at the initial instant, the vapor density be ρ_v^0 at this temperature, and the liquid density be ρ'_{liq}, respectively.

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From the condition of mass conservation in the tank we have

$$\eta_1 = \frac{1}{\rho'_{\text{liq}} - \rho'_{\text{v}}} [\rho'_{\text{liq}} - \overset{0}{\rho}_{\text{v}} \eta_0 - (1 - \eta_0) \overset{0}{\rho}_{\text{liq}}].$$

The mass of the evaporated liquid is

$$\Delta m = \frac{V}{\rho'_{\text{liq}} - \rho'_{\text{v}}} [\rho'_{\text{liq}} (\rho'_{\text{v}} - \overset{0}{\rho}_{\text{v}} \eta_0) - \rho'_{\text{v}} \overset{0}{\rho}_{\text{liq}} (1 - \eta_0)].$$

The increase in the temperature of the gas-liquid mixture is determined by the heat balance:

$$CV \overset{0}{\rho}_{\text{liq}} (1 - \eta_0)(T - T_0) + L\Delta m = qtS,$$

where q is a uniform and constant heat flux applied to the tank surface; S is the area of this surface.

The parameters of liquid and gaseous hydrogen along the saturation line [4] at pressures $p \simeq (1-5)\cdot 10^5$ Pa are such that $L\Delta m/CV(T - T_0) \lesssim 0.03$. Therefore the increase in temperature is determined by the formula

$$T_1 = T_0 + \frac{qSt}{CV \overset{0}{\rho}_{\text{liq}} (1 - \eta_0)}. \tag{1}$$

To calculate the pressure in the tank, we assume that at the beginning of the heating, the vapor-gas bubbles are filled with a supercharge gas at the pressure p_0 and a liquid vapor with the saturation pressure p_s at the liquid temperature T_0. Then, ignoring the charge in the amount of the supercharge gas dissolved in the liquid, we have the pressure at the end of the heating:

$$p_1 = p_s(T_1) + p_0 \frac{\eta_0 T_1}{\eta_1 T_0}.$$

Immiscible Liquid Model. Let us assume that the tank is filled with a stationary liquid with the temperature T_0 and a bubble is at the center. The heating of it by a constant heat flux q to the temperature of the onset of boiling at the wall T_b is described by an equation that is valid for an infinite cylinder [5]:

$$\Delta T = \frac{2qR}{\lambda} \left(\frac{1}{\sqrt{\pi}} \sqrt{\text{Fo}} + \frac{1}{4} \text{Fo} + \ldots \right),$$

where R is the cylinder radius. If the Fourier number $\text{Fo} = at/R^2 < 0.05$, then to an accuracy better than 10% we can ignore the curvature of the tank wall. For liquid hydrogen, for example, at $T \simeq 20$ K ($\lambda = 1.17 \cdot 10^{-3}$ W/(cm·deg), $a \simeq 2\cdot 10^{-3}$ cm²/sec) this condition implies $t \leq 10^6$ sec if the tank diameter is 3.8 m, and the increase in the temperature occurs by the law of heating of a semibounded body:

$$\Delta T = \frac{2q}{\lambda} (at/\pi)^{1/2}.$$

Estimating the conditions under which the curvature of the tank wall has a weak effect on the result of heating is of importance to us because it is in the approximation of one-dimensional ablation of a semibounded body that the boiling rate is estimated once the liquid starts to boil at the tank wall.

Pressure Growth in a Tank in the Case of Liquid Boiling at the Wall. Figure 1 shows the scheme of filling the tank with a gas and a liquid. In the center of the tank there is a bubble with volume V filled with a supercharge gas (helium) and gaseous hydrogen, which are in thermodynamic equilibrium with the adjacent liquid. When the liquid at the wall starts to boil as a result of heat inflow to the tank, an annular layer of gaseous hydrogen will

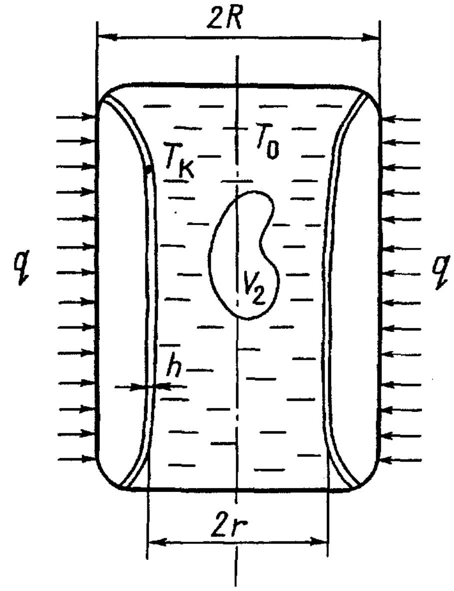

Fig. 1. Calculation scheme of the model of near-wall boiling of liquid hydrogen.

start to squeeze the liquid away from the wall, compressing the central bubble. The pressure in the tank will grow and the boiling temperature T_b will increase in accordance with the liquid saturation pressure.

We will relate the change in the bubble volume to the amount of evaporated liquid. To do this, we will resort to the relation between the initial V_1 and final V_2 volume of the central bubble:

$$V_1 = V_2 + \pi (R^2 - r^2) l - 2\pi R_{av} hl . \tag{2}$$

The equality of masses of the evaporated liquid and the vapor jacket yields

$$\pi \rho_v (R^2 - r^2) l = \rho_{liq} R_{av} 2\pi lh . \tag{3}$$

Assuming $R_{av} = (R + r)/2$, from (3) we have $R = r + \rho_{liq}/\rho_v h$. From (2) and (3) it follows that

$$h = R \frac{\rho_v}{\rho_{liq}} \left[1 - \sqrt{\left(1 - \frac{\rho_{liq} (V_1 - V_2)}{\pi R^2 l (\rho_{liq} - \rho_v)} \right)} \right] . \tag{4}$$

To avoid being engaged in complex details of the heat and mass transfer process that occur in compression of the central bubble but that are secondary for the given problem, we will determine the ratio V_2/V_1 under two limiting assumptions.

1. Compression occurs adiabatically, and in this case the pressure of the supercharge gas (helium) and the liquid vapor depends on V_2/V_1 as follows:

$$p = p_{He} (V_1/V_2)^{\gamma} + p_s (T_1) , \quad T_1 = T_0 \left(\frac{V_1}{V_2} \right)^{\gamma - 1} ,$$

where γ is the adiabatic exponent of the supercharge gas. This implies that the liquid-gas interface assumes the temperature of the supercharge gas, compressed adiabatically in the bubble, and the pressure of the liquid vapor corresponds to the temperature T_1.

2. Compression of the bubble occurs isothermally, i.e., the liquid mass with temperature T_0 around the bubble is considered to act as a thermostat for the bubble contents:

$$p = p_{He} \frac{V_1}{V_2} + p_s (T_0) .$$

In both cases the limiting permissible value is taken to be p. From the dependence of the pressure p_s and the density ρ_v we determine the temperature to which the boiling liquid is heated at the boundary of the vapor jacket and the vapor density ρ_v corresponding to it, which together with the value of V_1/V_2 is then used to determine h.

Computation of the Evaporation Time for a Liquid Layer of Thickness h. When h is known, the evaporation time can be determined using the model of ablation removal of material in melting [6]. For a steady process this rate is easily determined in a one-dimensional approximation:

$$u_0 = \frac{q}{L\rho_{liq} (1 + \varepsilon)} ,$$

where $\varepsilon = c/L(T_6 - T_0)$. However, at the beginning of the ablation process there is a nonstationary stage in which the rate $u < u_0$. As the calculations show, at the values of the heat flux of interest from units of W/m^2 to a hundred W/m^2 liquid hydrogen boiling fits entirely in the nonstationary stage, in which $u/u_0 < 0.92$. Therefore we will resort to the results of analyzing nonstationary ablation [6].

The dimensionless evaporation rate u/u_0 depends implicitly (in terms of the parameter y) on the dimensionless time $\theta = t/t_0$:

$$u/u_0 = (1 + \varepsilon) \frac{y}{1 + y} , \tag{5}$$

$$\ln (1 - \varepsilon y) + \frac{\varepsilon y}{1 + \varepsilon} = - \frac{\varepsilon^2 (\theta - 1)}{1 + \varepsilon} . \tag{6}$$

Integrating u/u_0 with respect to θ enables us to determine h:

$$h = u_0 t_0 (\theta - 1 - y) . \tag{7}$$

The time of heating of a semibounded body from T_0 to T_b by the heat flux q is taken as the time scale t_0:

$$t_0 = \frac{0.79 (T_b - T_0)^2 \lambda^2}{a q^2} . \tag{8}$$

The depth of warmup of the liquid δ is equal to :

$$\delta = (y + 1) \sqrt{\left(\frac{3 a t_0}{1 - 2/e} \right)} .$$

A special feature of the process of boiling of liquid hydrogen at the wall of a closed tank is that the phase transition temperature T_b does not remain constant but increases in accordance with the pressure in the tank. A change in T_b once the boiling starts implies a change in ε in Eqs. (5) and (6). Fortunately, at a hydrogen saturation pressure of $(1-5) \cdot 10^5$ Pa (and, correspondingly, the temperatures $T_b \simeq 20-30$ K) ε is small ($\varepsilon \simeq 0.01-0.025$). A change in ε within these limits alters h by less than 1%. Therefore for calculations we can assume $\varepsilon = 0.015$ to good accuracy and consider $\varepsilon \simeq$ const.

Example of Calculating the Pressure Growth in a Tank. As an example we will assume that the tank has a diameter of 3.8 m and a length $l = 6$ m. The initial temperature of the liquid hydrogen is $T_0 = 16$ K. The supercharge gas pressure is $1.02 \cdot 10^5$ Pa. The gaseous hydrogen pressure at $T = 16$ K is $p_s = 0.237 \cdot 10^5$ Pa. The liquid starts to boil when the temperature at the wall attains $T_b = 21$ K in the case of a total pressure of the mixture in the bubble (and in the entire tank) $p_{He} + p_s = 1.237 \cdot 10^5$ Pa. We will assume the filling of the tank with the

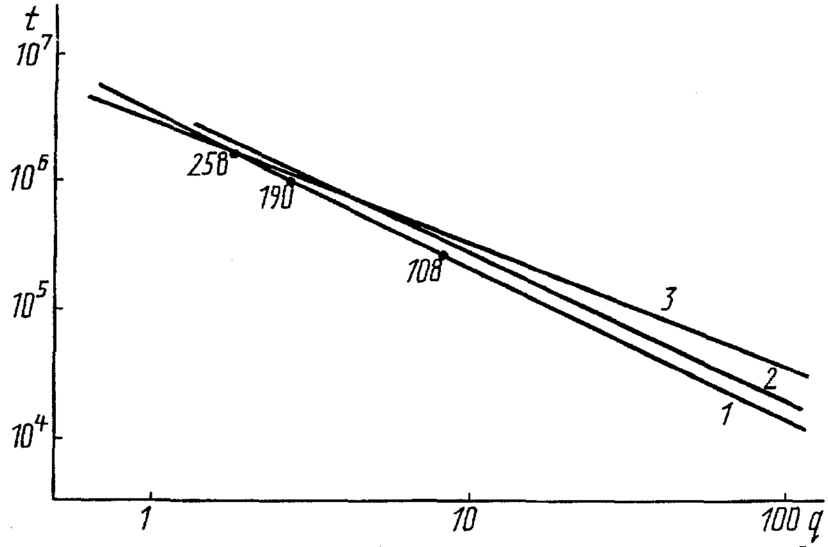

Fig. 2. Dependence of the time in which the pressure $p = 3.25 \cdot 10^5$ Pa is attained in the tank: 1) adiabatic compression of a central bubble; 2) isothermal compression; 3) compression by the complete mixing model. The figures on the curve are the depth of warmup of the liquid in centimeters. t, sec; q, W/m^2.

liquid $1 - \eta_0 = 0.8$, a bubble volume $V_1 = 13.6$ m^3, and a tank volume 68 m^3. The lateral surface through which the heat flux passes is equal to $S = 71.6$ m^2. In adiabatic compression of the central bubble to the pressure $p = 3.25 \cdot 10^5$ Pa the V_1/V_2 ratio is 1.5. In isothermal compression it is $V_1/V_2 = 2$.

From (4) it follows that in the first case $h = 0.378$ cm and in the second one $h = 0.568$ cm.

The time needed for evaporation is determined by Eq. (7) in the following manner. We prescribe $u/u_0 < 1$ and find y from (5) and θ from (6). Then from (7) we determine the value of q that enters both the time t_0 and rate u_0 scale:

$$q = \frac{0.79\,(T_b - T_0)^2 \lambda^2}{haL\rho_{\text{liq}}\,(1 + \varepsilon)}\,(\theta - 1 - y)\,.$$

Having determined q, from Eq. (8) we find the scale t_0 and the time of the process $t = \theta t_0$. By varying u/u_0, we obtain a number of values for t and q.

The results of the calculations are given in Fig. 2 for two cases: adiabatic and isothermal compression of the central bubble. The calculations show that the masses of the hydrogen boiled away at the wall differ by a factor of 1.5 and the times needed for this differ somewhat less (by a factor of 1.35–1.39). The dependence of t on q obtained by the model of complete mixing (Eq. (1)) is also given there. The plot shows that the complete mixing model yields a large time than the model of near-wall boiling but this conclusion is not valid at small q. This is associated with the fact that the heat penetration depth $\delta(t)$ for small heat fluxes (and correspondingly for times $t > 8 \cdot 10^5$ sec) becomes too large and even exceeds the tank radius R (for $t > 10^6$ sec). This implies that the model of ablation of a semibounded body becomes inapplicable here. To orient oneself regarding the applicability of the model, the plot shows heat penetration depths.

Conclusions. The proposed procedure for calculating the time in which the pressure in a tank with liquid hydrogen increases under zero gravity enables us to obtain upper and lower bounds of this time as a function of the amount of the heat flux q that goes directly to the liquid. The largest estimate is yielded by the model of complete mixing of the liquid while the smallest one is given by the model of adiabatic compression of a central gas bubble as a result of liquid boiling at the tank wall. True values of t lie somewhere between these two estimates. The considered numerical example of boiling of liquid hydrogen in a large tank shows that the model of near-wall boiling may yield overestimated times t for fairly small heat fluxes owing to the fact that it makes use of a model of ablation of a semibounded body that takes no account of the finite distance from the boiling surface to the central bubble. In specific calculations for small q when the heat penetration depth according to the calculations becomes larger than the distance to the central bubble, estimation performed by the complete mixing model seems more reliable if it yields a smaller time than the near-wall boiling model.

NOTATION

q, heat flux; T_0, initial temperature of the liquid; $\rho_{\rm liq}^0$ and $\rho_{\rm v}^0$, liquid and vapor densities at T_0; T_1, final temperature; $\rho'_{\rm liq}$ and $\rho'_{\rm v}$, liquid and vapor densities at the end of evaporation; η_0, fraction of the tank volume occupied by the gas-vapor mixture at the beginning of heating; η_1, the same, at the end of heating; Δm, mass of the evaporated liquid; V, tank volume; C, heat capacity of the liquid; L, heat of evaporation; p_0, supercharge gas pressure; p_s, hydrogen saturation pressure; S, lateral surface of the tank; λ, thermal condutivity of the liquid; a, thermal diffusivity of the liquid; R, radius of the tank; l, its length; h, thickness of the evaporated liquid layer; γ, adiabatic exponent; ε, dimensionless parameter of evaporation; u, evaporation rate; u_0, steady value of the evaporation rate; t_0, time scale; θ, dimensionless time; y, dimensionless parameter.

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